CMSC201
Computer Science I for Majors

Lecture 19 – Recursion

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Last Class We Covered

- Project 1 Details
- Classes
- Inheritance

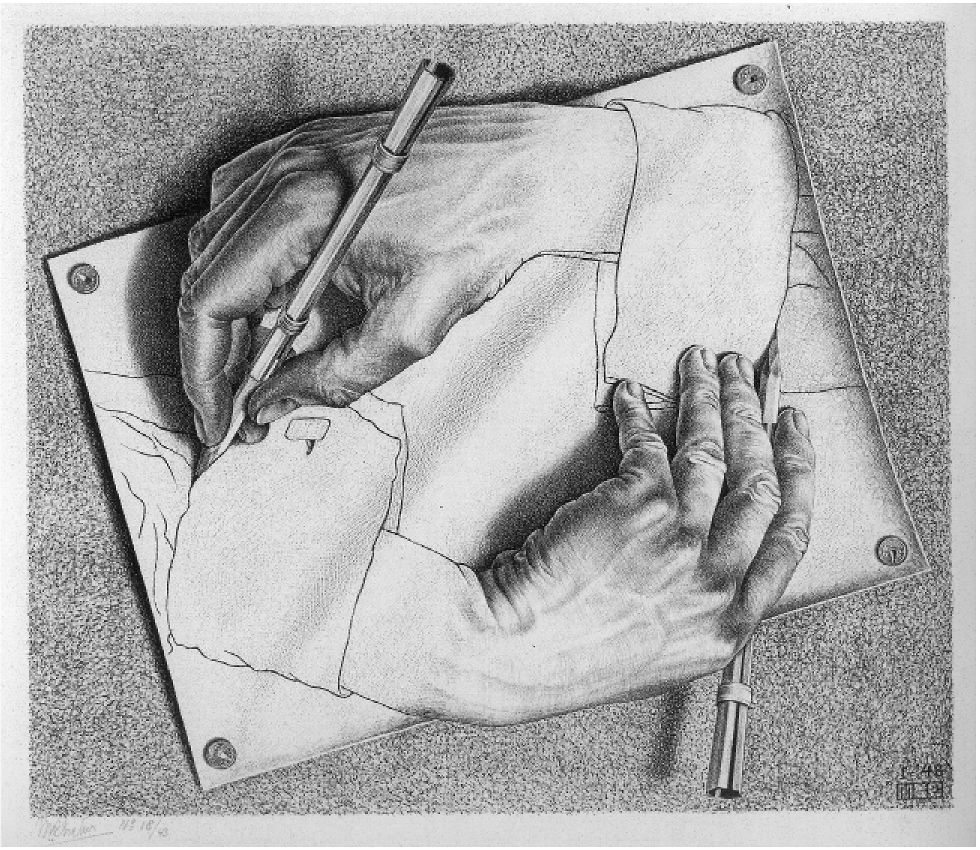
Any Questions from Last Time?

Today's Objectives

- To introduce recursion
- To begin to learn how to "think" recursively
- To better understand the concept of stacks

Introduction to Recursion

M.C. Escher: "Drawing Hands" (1948)



What is Recursion?

- In computer science, recursion is a way of thinking about and solving problems
- It’s actually one of the central ideas of CS

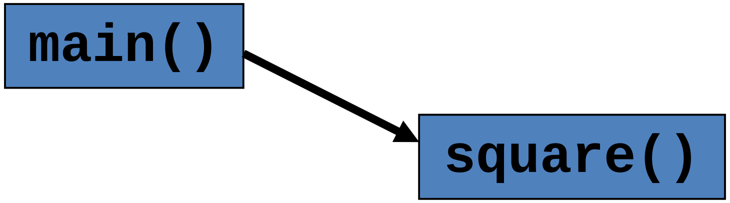
- Solving a problem using recursion means the solution depends on solutions to smaller instances of the same problem

Recursive Procedures

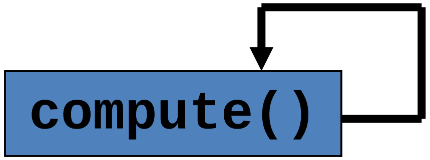
- When creating a recursive procedure, there are a few things we want to keep in mind:
  - We need to break the problem into smaller pieces of itself
  - We need to define a “base case” to stop at
  - The smaller problems we break down into need to eventually reach the base case

Normal vs Recursive Functions

- So far, we've had functions call other functions
  - For example, **`main()`** calls the **`square()`** function



- A recursive function, however, calls itself



Why Would We Use Recursion?

- In computer science, some problems are more easily solved by using recursive methods
- For example:
  - Traversing through a directory or file system
  - Traversing through a tree of search results
  - Some sorting algorithms recursively sort data
- For today, we will focus on the basic structure of using recursive methods

Simple Recursion Example

```
def compute(intInput):
    print(intInput)
    if (intInput > 2):
        compute(intInput-1)

def main():
    compute(50)

main()
```

This is where the recursion occurs.

You can see that the `compute()` function calls itself.

This program simply computes from 50 down to 2.

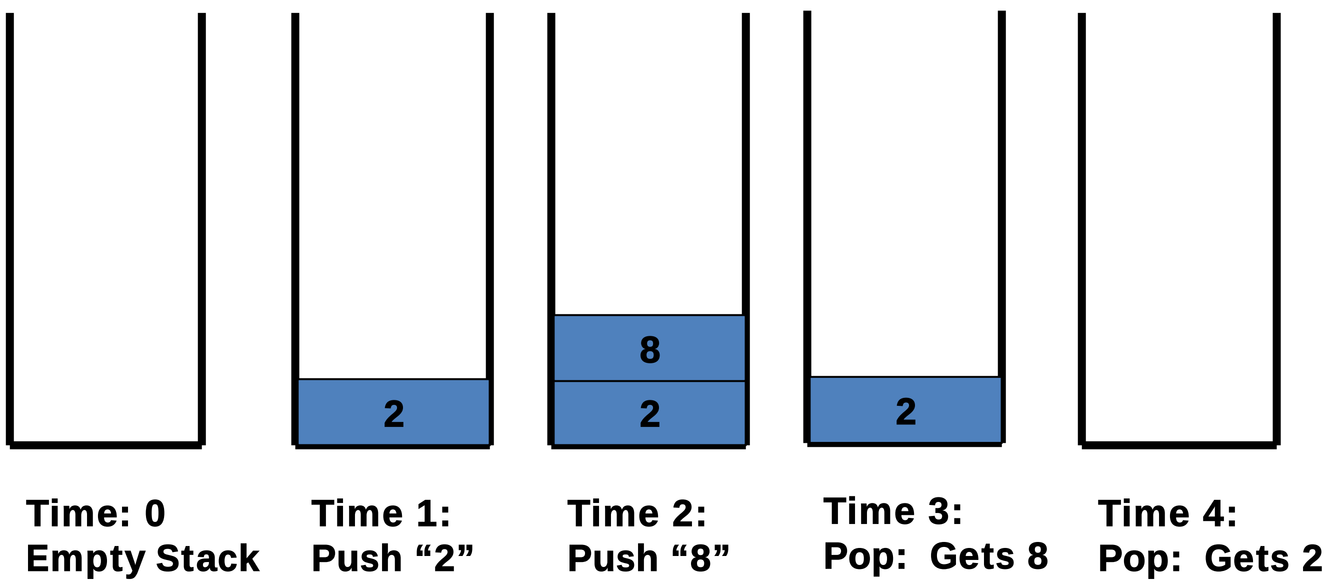
Visualizing Recursion

- To understand how recursion works, it helps to visualize what's going on.
- To help visualize, we will use a common concept called the *Stack*.
- A stack basically operates like a container of trays in a cafeteria. It has only two operations:
  - Push: you can push something onto the stack.
  - Pop: you can pop something off the top of the stack.
- Let's see an example stack in action.

Stacks

Stacks

- The diagram below shows a stack over time.
- We perform two pushes and two pops.



Stacks

- In computer science, a **stack** is a **last in, first out**(LIFO) abstract data type and data structure.
- A stack can have any abstract data type as an element, but is characterized by only two fundamental operations, the **push** and the **pop**.
- The push operation adds to the top of the list, hiding any items already on the stack, or initializing the stack if it is empty.

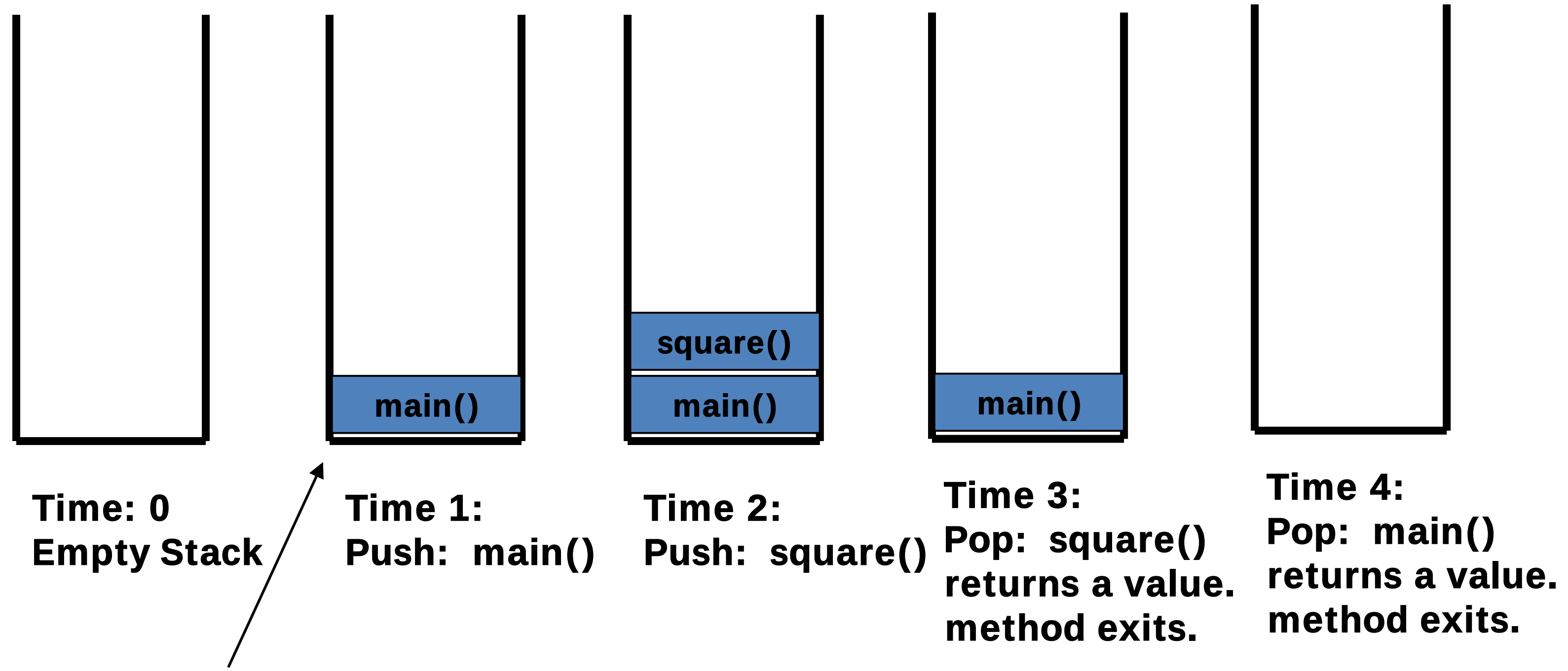
Stacks

- The nature of the pop and push operations also means that stack elements have a natural order.
- Elements are removed from the stack in the reverse order to the order of their addition: therefore, the lower elements are typically those that have been in the list the longest.

Stacks and Functions

- When you run a program, the computer creates a stack for you.
- Each time you invoke a function, the function is placed on top of the stack.
- When the function returns or exits, the function is popped off the stack.

Stacks and Functions



This is called an activation record or stack frame.

Usually, this actually grows downward.

Stacks and Recursion

- Each time a function is called, you *push* the function on the stack.
- Each time the function returns or exits, you *pop* the function off the stack.
- If a function calls itself recursively, you just push another copy of the function onto the stack.
- We therefore have a simple way to visualize how recursion really works.

Back to the Simple Recursion Program

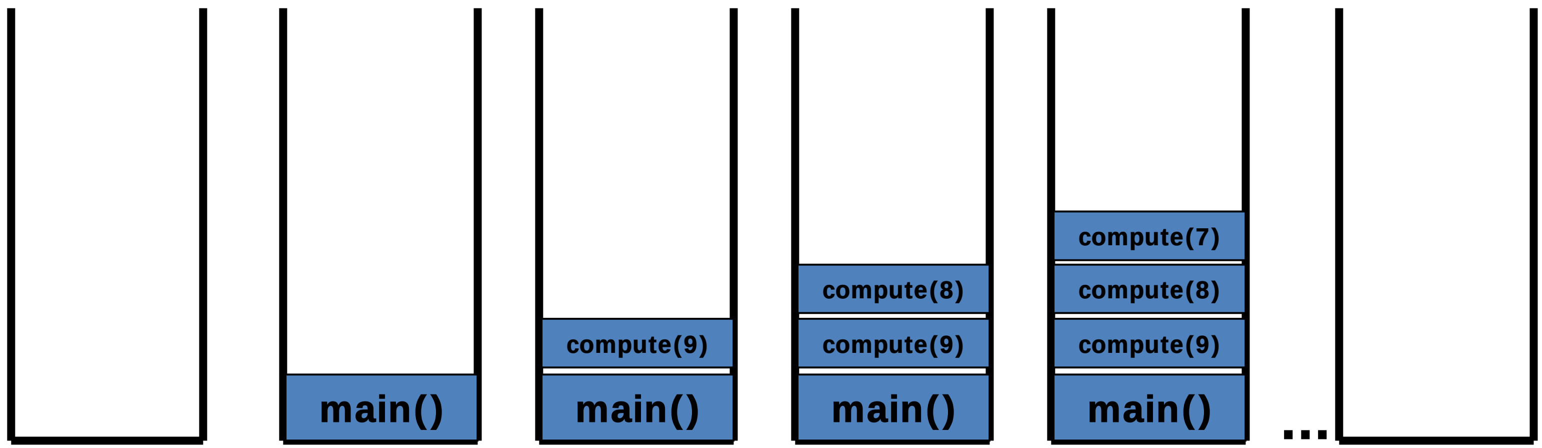
```
def compute(intInput):
    print(intInput)
    if (intInput > 2):
        compute(intInput-1)

def main():
    compute(50)

main()
```

Here's the code again. Now, that we understand stacks, we can visualize the recursion.

Stack and Recursion in Action



Time: 0
Empty Stack

Time 1:
Push: main()

Time 2:
Push: compute(9)

Time 3:
Push: compute(8)

Time 4:
Push: compute(7)

After returning from compute(2) pop everything

```
Inside compute(9):
print (intInput);        → 9
if (intInput < 2)
    compute(intInput-1);
```

```
Inside compute(8):
print (intInput);        → 8
if (intInput < 2)
    compute(intInput-1);
```

```
Inside compute(7):
print (intInput);        → 7
if (intInput < 2)
    compute(intInput-1);
```

Defining Recursion

Terminology

```
def f(n):
  if n == 1:
   return 1
  else:
   return f(n - 1)
```

base case

recursive case

"Useful" recursive functions have:

- at least one *recursive case*
- at least one *base case*
  so that the computation terminates

Recursion

```
def f(n):
  if n == 1:
   return 1
  else:
   return f(n + 1)
```

Find f(5)

We have a base case and a recursive case. What's wrong?

Recursion

The recursive case
should call the function
on a *simpler input*,
bringing us closer and closer
to the base case.

Recursion

```
def f(n):
  if n == 0:
    return 0
  else:
    return 1 + f(n - 1)
```

Find f(0)

Find f(1)

Find f(2)

Find f(100)

Recursion

```
def f(n):
   if n == 0:
    return 0
   else:
    return n + f(n - 1)

f(3)
3 + f(2)
3 + 2 + f(1)
3 + 2 + 1 + f(0)
3 + 2 + 1 + 0
6
```

Factorial

- $4! = 4 \times 3 \times 2 \times 1 = 24$

Factorial

- Does anyone know the value of 9?
- 362,880
- Does anyone know the value of 10?
- How did you know?

Factorial

- $9! = \quad 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- $10! = 10 \times \quad 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- $10! = 10 \times \quad 9!$
- $n! = n \times (n - 1)!$
- That's a recursive definition!

Factorial

```
def fact(n):
    return n * fact(n - 1)
```

fact(3)

3 × fact(2)

3 × 2 × fact(1)

3 × 2 × 1 × fact(0)

3 × 2 × 1 × 0 × fact(-1)

...

Factorial

- What did we do wrong?

- What is the base case for factorial?

Any Other Questions?

Announcements

- Lab has been cancelled this week!
  - Work on your project instead

- Project 1 is out
  - Due by Tuesday, November 17th at 8:59:59 PM
  - Do NOT procrastinate!

- Next Class: Recursion